Technische Universität München
Institut für Informatik
Theoretical Computer Science

WS 2017/18

Fundamental Algorithms 7

Exercise 1 (Hash Function)

Let $n = 1000$. Compute the values of the hash function $h(k) = \lfloor n(ak - \lfloor ak \rfloor) \rfloor$ for the keys $k \in \{61, 62, 63, 64, 65\}$, using $a = \frac{\sqrt{5}-1}{2}$. What do you observe?

Exercise 2 (Hash Table)

Let T by a hash-table of size 9 with the hash function $h : U \to \{0, 1, \ldots, 8\}, k \mapsto k \bmod 9$. Write down the entries of T after the keys 5, 28, 19, 15, 20, 33, 12, 17, and 10 have been inserted. Use chaining to resolve collisions.

Exercise 3 (Open Hash)

Now, let T be a hash table of size 11, using open addressing with the following hash functions

1. $h(k, i) := (k + i) \bmod 11$
2. $h(k, i) := \big(k \bmod 11 + 2i + i^2\big) \bmod 11$
3. $h(k, i) := (k \bmod 11 + i \cdot (k \bmod 7)) \bmod 11$

Insert the keys 5, 19, 27, 15, 30, 34, 26, 12, and 21 (in that order) and state which keys require the longest probe sequence in the resulting tables.

Exercise 4 (Hashing the Universe)

Consider a universe U of keys, where $|U| > mn$, and a hash function $h : U \to \{0, 1, \ldots, n - 1\}$. Show that there are at least m elements of U which are mapped to the same hash value, i.e. there is a subset A of U with $|A| = m$ and $h(a_1) = h(a_2)$ for all $a_1, a_2 \in A$.